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THE DAVID W. TAYLOR MODEL BASIN

AERODYNAMICS LABORATORY

WASHINGTON 7. D.C.

THEORY OF THE ANNULAR NOZZLE IN PROXIMITY TO THE GROUND

by

Harvey R. Chaplin

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July 1957

SYMBOLS

P_j	jet total pressure in pounds per square foot
p	ambient static pressure in pounds per square foot
$p + \Delta p$	nozzle base pressure in pounds per square foot
R	radius of curvature of the jet in feet, measured in a meridian plane of the jet
j	local momentum flux of the jet per foot (curvilinear) length in pounds per foot
h	nozzle altitude above ground in feet
ℓ	lift per foot length of a two-dimensional nozzle in pounds per foot
θ	angle of divergence of the jet from the nozzle axis in degrees
b	two-dimensional nozzle width (distance between the exits) in feet
A	augmentation factor; ratio of the total lift experienced by the nozzle to the total jet momentum flux
r	distance of a point on the jet sheet from the nozzle axis, in feet
L	total lift in pounds
J	total jet momentum flux in pounds
x	distance of a point on the jet sheet from the nozzle plane in feet
s	curvilinear distance of a point on the jet sheet from the nozzle exit in feet
F_b	force acting on the nozzle base in pounds
ρ	R/r_o
η	r/r_o

SYMBOLS (concl)

β $J/2F_b$

σ s/r_o

ξ x/r_o

$\bar{\xi}$ h/r_o

α angle between the nozzle axis and a line perpendicular to the ground in degrees

S   plan-view area of the nozzle base in square feet

C   perimeter of the nozzle base in feet

Subscripts

o   conditions at the nozzle exit

c   critical condition

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AERODYNAMICS LABORATORY
DAVID TAYLOR MODEL BASIN
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WASHINGTON, D. C.

THEORY OF THE ANNULAR NOZZLE IN

PROXIMITY TO THE GROUND

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SUMMARY

A simple theory is presented for the effect of ground proximity on the force acting on a stationary body containing a downward-blowing annular nozzle around the periphery of its base. The results are found to be in agreement with experimental data.

INTRODUCTION

Reference 1 presents experimental data showing that, when an annular nozzle is exhausted toward the ground from a short distance above the ground, a positive pressure builds up on the base area enclosed by the nozzle so that the body containing the nozzle experiences an upward force considerably greater than the thrust of the jet itself. A simple theory is presented in the present report to account for the essential features of this phenomenon.

ANALYSIS

The details of a flow involving an air jet with turbulent expansion and entrainment of secondary air are quite complex and have been satisfactorily approximated mathematically in only a few very special cases. The salient property of a jet, however, is its momentum; and many problems involving jets can be treated satisfactorily by taking account of the momentum only, neglecting the other features of the jet flow. This technique will be employed here: it will be assumed that the jet sheet is thin and nonmixing.

TWO-DIMENSIONAL CASE

Consider a plate of width b and infinite length to be situated above and parallel to the ground at an altitude h above it, with a jet of momentum j per unit length being exhausted vertically downward from each edge of the plate. If these jets stagnate on the ground, a pressure equal to the total pressure, p_j, of the jets will tend to build up in the space enclosed between them; but when the pressure inside this space, $p + \Delta p$, rises above the ambient pressure, p, the jets will curve outward along an arc of radius R defined by the balance of static and centrifugal pressures

$$\Delta p = j/R$$
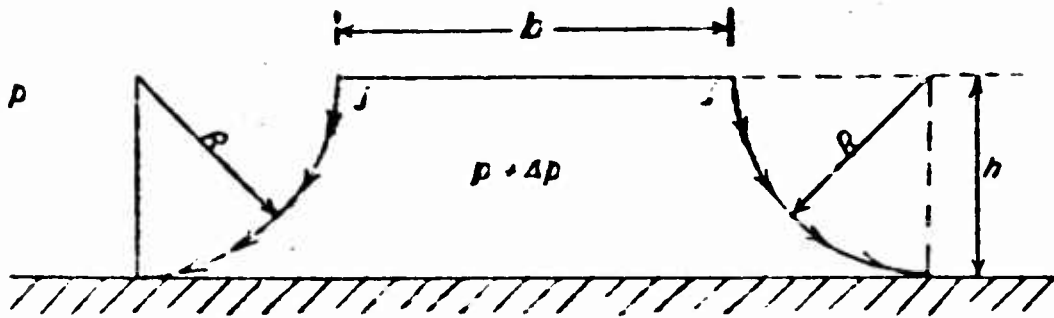
Assume that $(p_j - p) > \frac{1}{h}$. Now if $R > h$, there will be a stagnation point on the ground and Δp will tend to rise to

$$\Delta p = \frac{1}{R} = (p_j - p)$$

but this leads to a contradiction of the assumption

$$(p_j - p) > \frac{1}{h} > \frac{1}{R}$$

Further, if $R < h$, the space is no longer enclosed and Δp will fall to zero; but this leads to another contradiction, since $R \to \infty$ as $\Delta p \to 0$. Clearly, the only possibility is

$$R = h$$

in which case

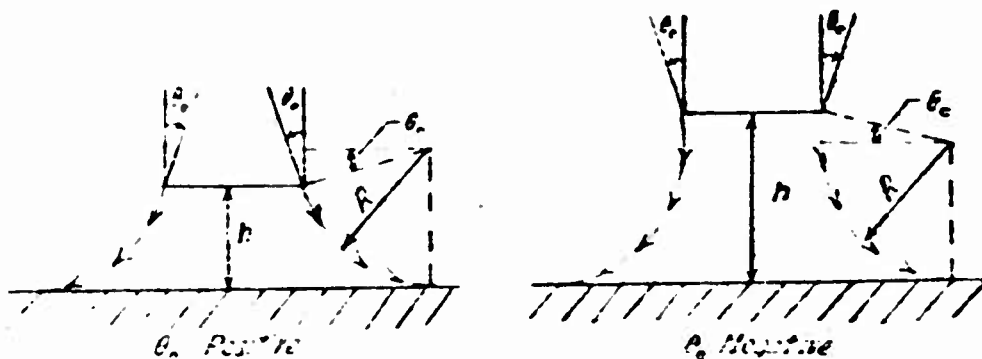$$\Delta p = \frac{1}{h}$$

The total lift force, $\mathcal{L}$, experienced by each unit length of the plate is

$$\mathcal{L} = 2j + \Delta p\, b$$

$$= 2j + jb/h$$

$$= 2j \left(1 + \frac{1}{2h/b}\right)$$

so that the primary thrust, 2j, of the jets is augmented by the factor $(1 + \frac{1}{2h/b})$. If the initial direction of the jets is not vertically down, but inclined at an angle θ_o from the vertical, then

$$\Delta p = \frac{1}{R} = \frac{1}{h}(1 - \sin \theta_o)$$

θ_o Positive θ_o Negative

The total lift now becomes

$$\ell = 2j \cos \theta_o + \frac{j(1 - \sin \theta_o)}{h} b$$

and the augmentation factor is

$$A = \frac{\ell}{2j} = \cos \theta_o + \frac{1 - \sin \theta_o}{2h/b}$$

This factor is a maximum when

$$\theta_o = \theta_{o_{opt}} = -\tan^{-1}\left(\frac{1}{2h/b}\right)$$

in which case

$$\left(\frac{\ell}{2j}\right)_{max} = \frac{1}{\sqrt{1 + \frac{1}{(2h/b)^2}}} + \frac{1 + \frac{1}{\sqrt{1 + (2h/b)^2}}}{2h/b}$$

The augmentation factor, together with the values of $\theta_{o_{opt}}$, is plotted in Figure 1 for $\theta_o = 0$ and $\theta_o = \theta_{o_{opt}}$. It is seen that a considerable advantage accrues from operating the jets at the optimum angle. For example, with the plate at an altitude of one-quarter of the plate width the augmentation factor is 4.24 with the jets at the optimum angle of -63.4° as compared to 3.00 with the jets vertical. The question of jet stability should be mentioned at this point. Reference 1 reported that when the annular nozzle tested was raised above a certain height from the ground, the annular jet "collapsed" into a round jet before striking the ground, in which case the base experienced a negative Δp, making the augmentation ratio less than unity. This phenomenon is certain to be influenced by the jet angle, θ_o, and may have an important bearing on the extent to which the theoretical advantage of operating at optimum θ_o can be realized. This question will be discussed in a later section, but further experimental study will be required to arrive at definite, reliable conclusions.

It might also be well at this point to consider briefly the difference between the ideal jet flow (thin, nonmixing jet) which has been assumed, and the flow which will exist in a real situation. The principal difference arises from the tendency of the jet to entrain the surrounding air into itself. Thus, whereas the ideal jet becomes tangent to the ground without a

stagnation point, the real jet will impinge at a shallow angle
so that a part of the jet air is fed back into the cavity to
replace the air which is entrained from the cavity.

Ideal Flow   Real Flow
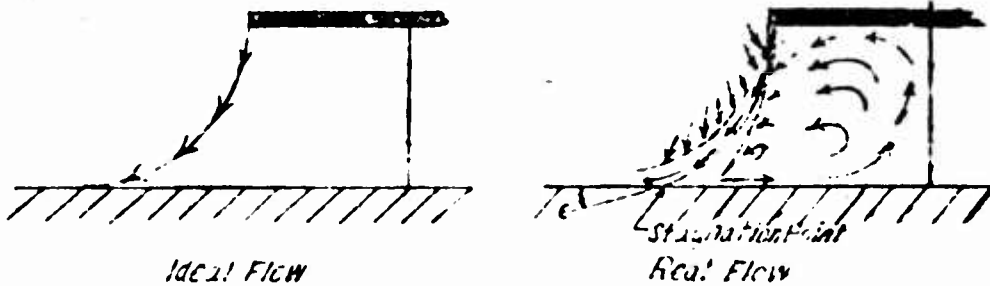
The practical difference between the real and ideal flows, in
terms of the relation between the augmentation ratio and h/b,
will be negligible so long as the impingement angle, ϵ, is
small.

AXIALLY SYMMETRIC CASE

Consider a disc of radius r_o to be situated above and
parallel to the ground at an altitude h above it, with an
annular jet of total momentum J being exhausted downward from
the perimeter of the plate.
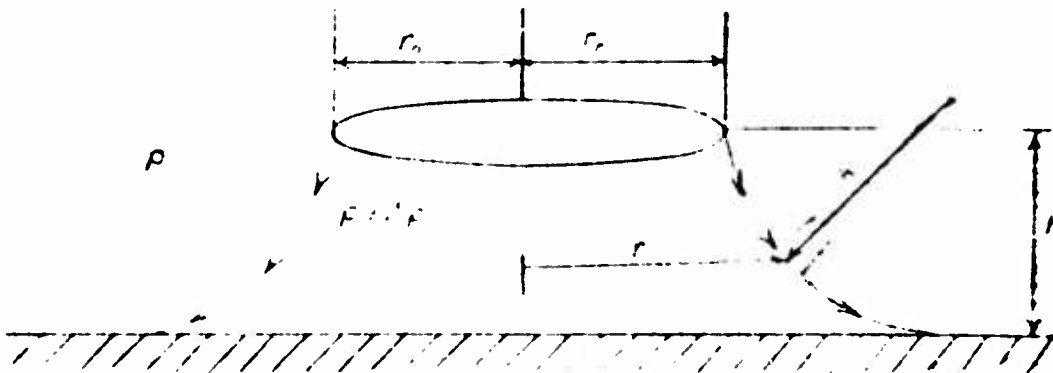
As in the two-dimensional case, a positive pressure will build up in the cavity formed by the disc, the jet, and the ground; and the jet will curve outward until it becomes tangent to the ground.

ANALYTICAL APPROXIMATION -- As before, the radius of curvature, R, is defined by

$$\Delta p = j/R$$

where j is the momentum flux per unit "length" (length being measured along a horizontal circumference) of the jet; that is,

$$j = J/2\pi r$$

Defining the initial jet strength

$$j_o = \frac{J}{2\pi r_o} ,$$

then

$$j = j_o \frac{r_o}{r}$$

whence

$$R = \frac{j_o}{\Delta p} \cdot \frac{r_o}{r}$$

The equivalent expression in the two-dimensional case is

$$R = \frac{j}{\Delta p}$$

where, in this case, j is constant along the jet. Therefore, it is possible to apply the two-dimensional results (with appropriate rescaling) directly to the axially symmetric problem under conditions such that, along the free surface of the jet,

$$r_o/r \approx 1$$

This condition is strictly satisfied if

$$h/r_o \ll 1$$

but the approximate solution will actually give reasonable
answers over an important part of the range of practical interest.
Under this approximation,

$$\Delta p \approx \frac{J}{2\pi r_o R}$$

$$R \approx h/(1 - \sin \theta_o)$$

Defining

$$F_b = \pi r_o^2 \, \Delta p$$

$$2F_b \approx J \frac{r_o}{h} (1 - \sin \theta_o)$$

The total lift is given by

$$L = J \cos \theta_o + F_b$$

$$\approx J \left(\cos \theta_o + \frac{1 - \sin \theta_o}{2h/r_o} \right)$$

The augmentation factor is

$$A = L/J \approx \left(\cos \theta_o + \frac{1 - \sin \theta_o}{2h/r_o} \right)$$

and the optimum jet angle is

$$\theta_{o_{opt}} \approx -\tan^{-1} \left(\frac{1}{2h/r_o} \right)$$

Figure 1 thus gives the approximate augmentation for the axially symmetric case directly, if the abscissas are interpreted to be values of h/r_o rather than of h/b.

GRAPHICAL APPROXIMATION -- The radius of curvature of the jet sheet in a meridian plane in the axially symmetric case was given as

$$R = \frac{J_o}{\Delta p} \frac{r_o}{r}$$

This can be rewritten

$$R = \frac{J}{2\pi r_o \Delta p} \frac{r_o}{r}$$

$$R/r_o = \frac{J}{2F_b} \frac{r_o}{r}$$

$$\rho = \beta \frac{1}{\eta}$$

where

$$\rho = R/r_o$$

$$\eta = r/r_o$$

$$\beta = \frac{J}{2F_b} = \frac{1}{2(A - \cos \theta_o)}$$

Given the augmentation factor and θ_o, the dimensionless altitude, $\xi = h/r_o$, can be determined approximately by a very simple graphical procedure. While this procedure becomes rather laborious if an accurate answer is sought, its consideration facilitates a clear understanding of both the exact solution,

derived in the next section, and the conditions under which
the approximate solution is acceptable.

The graphical solution is made possible by dividing η
into increments and assuming that the part of the jet path
lying within a given increment is satisfactorily represented
by an arc of radius

$$\rho_m = \frac{\beta}{\eta_m}$$

where η_m is the mean value of η within the increment; that is,
the value of η at the center of the increment. The solution
may be made as accurate as one pleases, by taking the incre-
ments of η sufficiently small.

The procedure is illustrated in Figure 2, for the case
$\theta_o = -\pi/4$, augmentation factor = 2.41, taking increments of
0.05 in η. The jet path corresponding to the analytical
approximation is also shown in Figure 2 for comparison. Note
that the analytical approximation is equivalent to choosing
one large increment of η (having its center at $\eta = 1$) which
includes the entire free path of the jet. The graphical approx-
imation is thus the more exact of the two.

Some refinements of the procedure outlined above are
obvious, but will not be discussed since the graphical approx-
imation is not proposed as a practical technique.

EXACT SOLUTION -- The equations defining the path of the jet
are

$$\frac{d\theta}{ds} = \frac{1}{R} = \frac{2\mathit{F}_b}{J} \cdot \frac{r}{r_o^2}$$

$$r = r_o + \int_0^s \sin\theta\, ds$$

where θ is the angle of inclination of the jet at any point
measured (positive outward) from the vertical in a meridian
plane, and s is the curvilinear distance from the nozzle to
the point measured along the jet in a meridian plane.
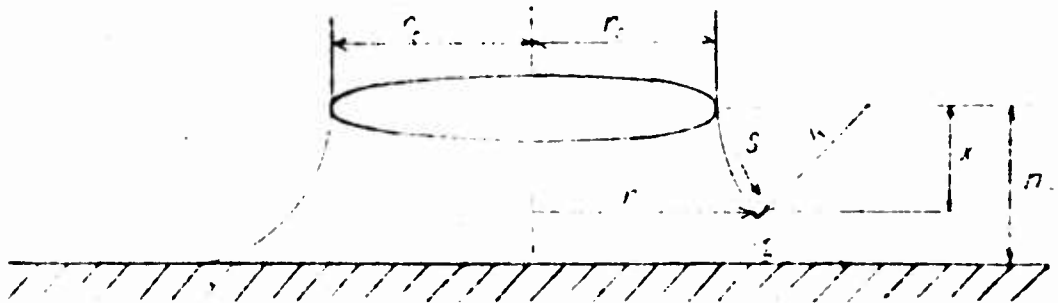
In addition to the dimensionless quantities ρ, η, β, already
defined, the following parameters will be introduced:

$$\xi = \frac{x}{r_o}$$

$$\sigma = \frac{s}{r_o}$$

$$\bar{\xi} = \frac{h}{r_o}$$

The equations now become

$$\frac{d\theta}{d\sigma} = \frac{1}{\beta} \eta$$

$$\eta = 1 + \int_0^\sigma \sin\theta \, d\sigma$$

The solution sought is the relation between β, θ_0, and $\bar{\xi}$. The dimensionless altitude, $\bar{\xi}$, is related to the above system of equations by:

$$\xi = \int_0^\sigma \cos\theta \, d\sigma$$

$$\bar{\xi} = \int_{\theta=\theta_0}^{\theta=\pi/2} \cos\theta \, d\sigma(\theta)$$

Differentiating the first two dimensionless equations above with respect to σ gives

$$\frac{d^2\theta}{d\sigma^2} = \frac{1}{\beta} \frac{d\eta}{d\sigma}$$

$$\frac{d\eta}{d\sigma} = \sin\theta$$

These equations combine to the single equation

$$\frac{d^2\theta}{d\sigma^2} = \frac{1}{\beta} \sin\theta$$

Multiplying both sides by $2 \frac{d\theta}{d\sigma} d\sigma$ gives

$$2 \frac{d\theta}{d\sigma} \frac{d}{d\sigma} \left(\frac{d\theta}{d\sigma}\right) d\sigma = \frac{2}{\beta} \sin\theta \, d\theta$$

Integrating,

$$\left(\frac{d\theta}{d\sigma}\right)^2 = -\frac{2}{\beta}\cos\theta + const.$$

Evaluating the constant from the initial conditions,

$$at \ \sigma = 0, \ \frac{d\theta}{d\sigma} = \frac{1}{\beta}, \ \theta = \theta_0$$

gives

$$\left(\frac{d\theta}{d\sigma}\right)^2 = -\frac{2}{\beta}\cos\theta + \frac{1}{\beta^2} + \frac{2}{\beta}\cos\theta_0$$

$$\frac{d\theta}{d\sigma} = \sqrt{\frac{2}{\beta}}\ \sqrt{\cos\theta_0 + \frac{1}{2\beta} - \cos\theta}$$

$$d\sigma = \sqrt{\frac{\beta}{2}}\left(\frac{d\theta}{\sqrt{\cos\theta_0 + \frac{1}{2\beta} - \cos\theta}}\right)$$

The expression for $\bar{\xi}$ can now be introduced to give

$$\bar{\xi} = \int_0^{(\sigma)_{\theta=\pi/2}} \cos\theta \ d\sigma$$

$$= \sqrt{\frac{\beta}{2}} \int_{\theta_0}^{\pi/2} \frac{\cos\theta \ d\theta}{\sqrt{\cos\theta_0 + \frac{1}{2\beta} - \cos\theta}}$$

This is the desired solution, reduced to quadratures. Unfortunately, the result of the integration indicated cannot be expressed in closed form using elementary functions. The integration is readily performed, however, by any of several numerical procedures.

It will be recalled from the approximate analyses that the optimum value of θ_0 is always negative. Confining our attention,

then, to the range

$$\theta_o \leq 0$$

it will be noted that the integral has a real value only if

$$(\cos \theta_o + \frac{1}{2\beta}) \geq 1$$

This arises from the fact that, if $(\cos \theta_o + \frac{1}{2\beta}) < 1$, the theory predicts that the jet will converge on the axis at the angle

$$\theta = \cos^{-1} (\cos \theta_o + \frac{1}{2\beta})$$

This result, implying a stagnation point in the jet on the axis, is inconsistent with the mathematical model assumed and is without interest in the present discussion.

Note that the term $(\cos \theta_o + \frac{1}{2\beta})$ is equal to the augmentation ratio, A:

$$A = L/J = \cos \theta_o + \frac{1}{2\beta}$$

The solution can thus be rewritten

$$\bar{\xi} = \frac{1}{2\sqrt{A - \cos \theta_o}} \int_{\theta_o}^{\pi/2} \frac{\cos \theta \, d\theta}{\sqrt{A - \cos \theta}}$$

In the limiting case, $A = 1$, the jet will converge on the axis asymptotically, so that

$$\text{as } A \rightarrow 1 \quad , \quad \bar{\xi} \rightarrow \infty$$

This result is of academic interest, at least, since it shows that the ideal axially symmetric jet (unlike the two-dimensional case) will support a finite positive Δp (when $\theta_o < 0$) even at great altitudes.

It will be noted that the integrand,

$$\frac{\cos \theta}{\sqrt{A - \cos \theta}}$$

is symmetrical about $\theta = 0$.

Therefore,

$$\int_{\theta_0}^{\pi/2} \frac{\cos \theta \, d\theta}{\sqrt{A - \cos \theta}} = \int_0^{\pi/2} \frac{\cos \theta \, d\theta}{\sqrt{A - \cos \theta}} + \int_0^{-\theta_0} \frac{\cos \theta \, d\theta}{\sqrt{A - \cos \theta}}$$

Thus the labor involved in evaluating $\bar{\xi}$ can be reduced to negligible proportions by tabulating a single function, say

$$g(A, \theta) = \int_0^\theta \frac{\cos \theta \, d\theta}{\sqrt{A - \cos \theta}} \qquad \left\{ \begin{array}{l} 0 \le \theta \le \pi/2 \\ A \ge 1 \end{array} \right.$$

$$G(A) = \int_0^{\pi/2} \frac{\cos \theta \, d\theta}{\sqrt{A - \cos \theta}}$$

The desired solution would then be simply,

$$\bar{\xi} = \frac{1}{2\sqrt{A - \cos \theta_0}} \left[G(A) + g(A, -\theta_0) \right] \qquad \theta_0 < 0$$

$$\bar{\xi} = \frac{1}{2\sqrt{A - \cos \theta_0}} \left[G(A) - g(A, \theta_0) \right] \qquad \theta_0 > 0$$

Values of the required function are presented for a few values of A in Figure 3.

Figure 4 presents some results from the exact theory in the form of constant-augmentation contours, as functions of

θ_o and h/r_o, and plots of augmentation factor versus h/r_o for $\theta_o = 0$ and $\theta_o = \theta_{o_{opt}}$. Augmentation factor curves according to the approximate theory and according to an approximate analysis (see Appendix) of experimental results from Reference 1 are also presented, for comparison. It is seen that the theory is in reasonable agreement with experiment and that the approximate and exact solutions do agree satisfactorily at low values of h/r_o.

The experimental curve in Figure 4 happens to agree almost exactly with the approximate-theoretical curve. This experimental curve was, itself, derived from Reference 1 (Appendix A) by an approximate method, however, and it should not be concluded that the approximate theory will, in general, give more realistic results than the exact theory.

EFFECT OF INCIDENCE

If the stationary nozzle is not parallel to the ground, but tilted at an angle α to the ground, the flow picture becomes somewhat altered.

To avoid confusing the definitions of parameters, the nozzle will be considered to be horizontal and the ground to be inclined at an angle α. The flow pattern and aerodynamic forces are, of course, the same in this case as if the ground were horizontal and the nozzle inclined at an angle α.
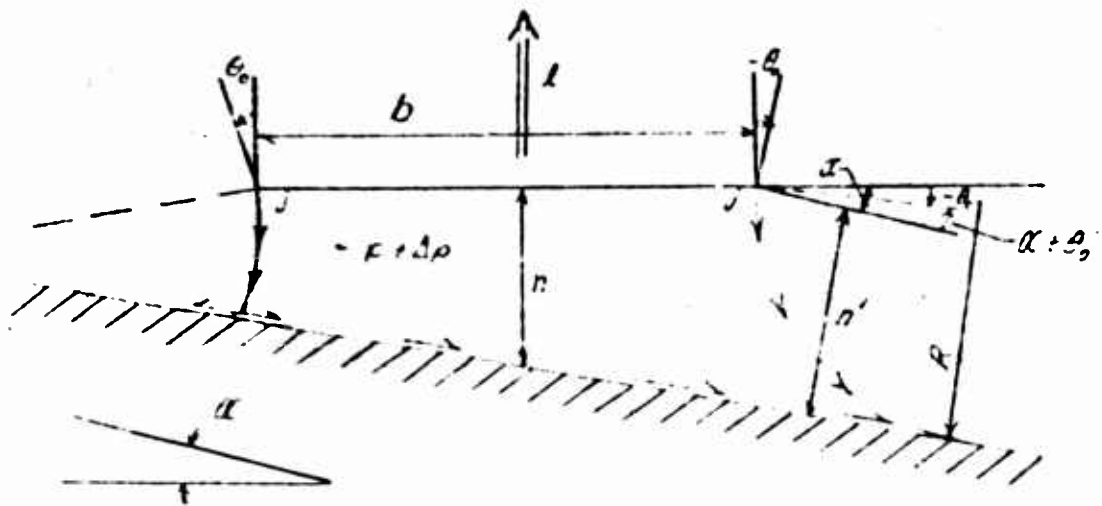
TWO-DIMENSIONAL CASE -- It is apparent from the sketch that if the jets are of equal strength and angle, and the pressure $(p + \Delta p)$ is constant within the cavity, it is no longer possible for both jets to come tangent to the ground. The pressure $(p + \Delta p)$ will adjust itself until the jet from the "high" side of the plate comes tangent to the ground, and the jet from the "low" side will impinge on the ground and split into two parts, the inward-deflected part eventually merging with the jet from the high side. The pressure rise, Δp, in the cavity is thus defined by conditions at the high side of the plate:

$$\Delta p = j/R$$

$$h' = R\left[1 - \sin(\alpha + \theta_o)\right]$$

$$h' = h \cos \alpha + b/2 \sin \alpha$$

$$R = \frac{h\cos \alpha + b/2 \sin \alpha}{1 - \sin(\alpha + \theta_o)}$$

$$\Delta p = \frac{1}{h} \frac{1 - \sin(\alpha + \theta_0)}{\cos \alpha + (b/2h) \sin \alpha}$$

The augmentation factor is thus

$$\frac{\ell}{2j} = \cos \theta_0 + \frac{1 - \sin (\alpha + \theta_0)}{(2h/b) \cos \theta + \sin \alpha}$$

Since the same result applies no matter which side of the plate is high, α is always taken to be positive.

AXIALLY SYMMETRIC CASE -- A rather complicated situation arises when the axially symmetrical annular nozzle is placed above a ground at incidence. If it is assumed that a uniform pressure rise will occur in the space enclosed between the jet, disc, and ground, it is seen that the pressure rise is limited to that Δp which the most vulnerable part of the jet (that is, the part of the jet originating at the highest point of the nozzle) is capable of sustaining. If this part of the jet comes tangent to the ground, all of the other parts of the jet will impinge on the ground at a finite angle and split, part of the jet leaving the point of impingement in an outward direction along the ground, and part inward. Whereas this inward motion of part of the jet created no special problem in the two-dimensional case, it has disturbing implications in the present case, since the inward-directed part of the jet is converging on itself. If the assumption of a thin, nonmixing (inviscid) jet were maintained, an extremely complex flow would establish itself inside

the cavity, and the assumption of constant pressure inside the cavity would be contradicted. The mathematical difficulty can be resolved by admitting viscous action between the jet and the ground; the initially thin, high-speed jet will then lose its momentum after only a short contact with the ground, and the assumption of constant pressure within the cavity can be justified. This pseudo-ideal jet flow, which manifests no viscous interaction with the air past which it flows in its free path, but which does manifest viscous action after contact with the ground, is certainly a closer approximation to the real flow, in the case of incidence, than the completely ideal jet flow considered up to now. It must be acknowledged, however, that, in the case of a real annular jet at incidence, a very considerable quantity of air will be recirculating through the cavity and escaping along the ground under the most vulnerable part of the jet. The theoretical solutions cannot, therefore, be expected to give equally reliable estimates of the real behavior as in the case of zero incidence.

As in the case $\alpha = 0$, an approximate solution for the axially symmetric case, valid at slight incidences and at low altitudes, can be immediately extracted from the two-dimensional solution. The result is

$$\Delta p \approx \frac{J}{2 \pi r_o} \cdot \frac{1 - \sin (\alpha + \theta_o)}{h \cos \alpha + r_o \sin \alpha}$$

$$A \approx \cos \theta_0 + \frac{1 - \sin (\alpha + \theta_0)}{2h/r_0 \cos \alpha + 2 \sin \alpha}$$

If the exact solution is desired, the following rule is applied in lieu of generalizing the previous exact solution:

The augmentation factor, A, for an annular nozzle with initial jet angle θ_0, at an altitude h/r_0 above a ground inclined at an angle α, bears the following relation to the augmentation ratio A' for an annular nozzle with initial jet angle $(\theta_0 + \alpha)$ at an altitude $(h/r_0 \cos \alpha + \sin \alpha)$ above a level ground:

$$A = A' - \cos(\theta_0 + \alpha) + \cos \theta_0$$

or, stated more concisely,

$$A(\theta_0, \bar{\xi}) = A'(\theta_0', \bar{\xi}') - \cos \theta_0' + \cos \theta_0$$
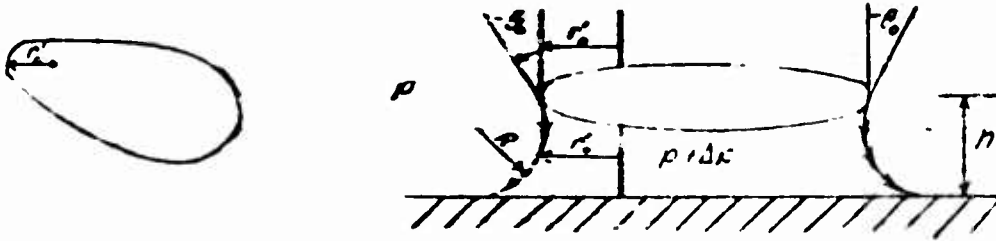
where
$$\theta_0' = \theta_0 + \alpha$$

$$\bar{\xi}' = h'/r_0 = \bar{\xi} \cos \alpha + \sin \alpha$$

Theoretical curves of augmentation factor versus h/r_0 for $0°$ and $15°$ incidence are compared in Figure 5 to curves derived from the experimental results of Reference 1. It is seen that the theoretical curves are in qualitative agreement with experiment, but the quantitative agreement is not as good with incidence as without.

"ANNULAR" NOZZLE OF ARBITRARY PLAN FORM

The approximate method of calculation can be applied immediately to a nozzle of arbitrary plan form.

As before, the balance of static and centrifugal pressures at a point on the free jet is given by,

$$\Delta p = \frac{j_0}{R} \frac{r_0'}{r'}$$

where r_0' is the local radius of curvature of the nozzle in plan view, and r' is the horizontal distance of the point from the vertical axis through the local center of curvature of the nozzle.

If
$$\frac{h}{r_0'} \ll 1 \quad .$$

then
$$\Delta p \approx \frac{j_0}{R}$$

If the jet is of uniform strength at the nozzle exit, then $j_0 = J/C$ where C is the perimeter of the nozzle, and

$$A \approx \cos \theta_0 + \frac{1 - \sin \theta_0}{hC/S}$$

where S is the area of the base enclosed by the nozzle. Note, however, that the restriction

$$h/r_0' \ll 1$$

is a very strong restriction indeed for plan forms where the

minimum radius of curvature is a small fraction of the maximum
dimension of the plan form.

Note that, according to the above approximate expression,
if nozzles of different shapes, but of the same plan-form area
and at the same altitude, are compared on the basis of augmenta-
tion, the optimum shape will be circular, since the circle has
the least possible perimeter for a given area.

When an exact solution is attempted for an ideal jet of
arbitrary plan form, a serious difficulty arises. Consider,
for example, the simple case of an oval consisting of a rectangle
and semicircular ends.
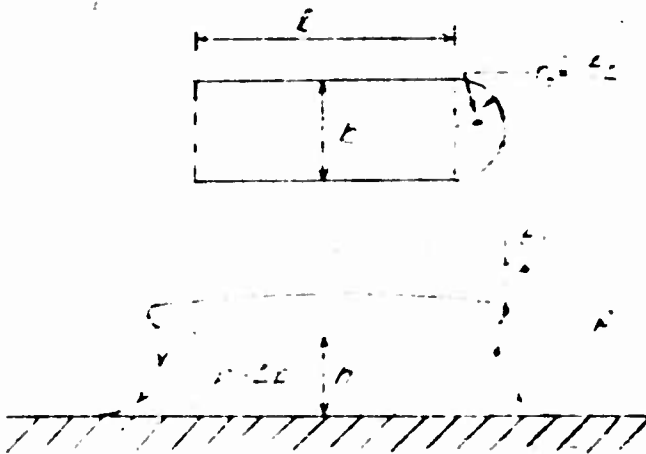
If, as before, a constant pressure (of sufficient magnitude to
bend some part of the jet to tangency with the ground) is assumed
to exist in the cavity, the parts of the jet exhausting from
the sides of the oval will behave exactly as in the two-dimensional
case, and the parts exhausting from the ends of the oval will
behave exactly as in the axially symmetric case.

If the strengths of the parts of the jet exhausting from the straight and curved parts of the nozzle are adjusted to j_{o1} and j_{o2}, respectively, in such a way that all parts of the jet come tangent to the ground, then

$$\Delta p = \frac{j_{o1}}{b/2} \, (A_1 - \cos \theta_o) = \frac{j_{o2}}{r_o/2} \, (A_2 - \cos \theta_o$$

$$\frac{j_{o1}}{j_{o2}} = 2 \, \frac{A_2 - \cos \theta_o}{A_1 - \cos \theta_o}$$

where A_1 and A_2 are the augmentation factors for a two-dimensional jet at dimensionless altitude h/b and for an axially symmetric jet at dimensionless altitude h/r_o, respectively, (taken, for example, from Figures 1 and 4, respectively). The augmentation factor for the oval is, in this case

$$A = \frac{jb + \pi r_o^2}{\dfrac{jb}{A_1 - \cos \theta_o} + \dfrac{\pi r_o^2}{A_2 - \cos \theta_o}} + \cos \theta_o$$

It is possible to work out similar "exact" solutions for many plan forms bounded by straight lines and arcs, but none of these solutions are strictly valid. In the example of the oval, if one examines the jet path in a vertical plane through the diameter of one of the semicircles, the jet sheet is found to be discontinuous. The shape of the jet path is slightly different for the "two-dimensional" and "axially symmetric"

parts of the jet, and there will be a crack in the surface of the ideal jet, which is inconsistent with the assumption of a uniform positive pressure within the cavity. Such inconsistencies arise whenever there are discontinuities in the (plan view) radius of curvature of the nozzle.

Despite its lack of strict validity, it is likely that this process of patching together exact solutions has a certain degree of practical validity and represents an improved approximation. The real jet has diffusive and cohesive properties, due to the turbulent mixing, which will tend to resist the formation of "cracks" in the jet sheet. No doubt the appearance of a minute crack in the ideal jet sheet indicates a slight weakening of the corresponding real jet sheet, but this effect should be small so long as the ratio $h/r'_{o_{min}}$ ($r'_{o_{min}}$ being the minimum plan-view radius of curvature of the nozzle) remains within reasonable bounds, say less than one.

This improved approximation can be summarized as follows:

1. The distribution of local jet strength along the nozzle should be

$$j''_o = \frac{\Delta p \ r'_o}{2(A'' - \cos \theta_o)}$$

$$j''_o \longrightarrow \frac{\Delta p h}{1 - \sin \theta_o} \quad \text{as} \quad \frac{h}{r'_o} \longrightarrow 0$$

where A'' is the augmentation factor for an axially symmetric nozzle at dimensionless altitude h/r_o' according to the exact solution.

2. The augmentation factor for the nozzle under consideration is then given to an improved approximation by

$$A \approx \cos \theta_o + \frac{\Delta p\, S}{J''}$$

where J'' is the total jet momentum, obtained by integrating the local jet strength along the circumference:

$$J'' = \int_C \frac{\Delta p\, r_o'}{2(A'' - \cos \theta_o)}\, dC$$

With this substitution,

$$A \approx \cos \theta_o + S \bigg/ \int_C \frac{r_o'}{2(A'' - \cos \theta_o)}\, dC$$

3. This improved approximate solution should be satisfactory for reasonably small altitudes compared to the minimum radius of curvature of the plan form, say

$$\frac{h}{r_{o_{min}}'} < 1$$

If the plan-view radius of curvature of the nozzle varies continuously along the circumference, as, for example, in the case of an elliptical nozzle, this restriction on $h/r_{o_{min}}'$ may be relaxed and the "improved-approximate" procedure indicated above yields an exact solution.

STABILITY OF THE JET — CRITICAL ALTITUDE

It was shown that, in the ideal case, the augmentation factor for an axially symmetric annular nozzle parallel to the ground with θ_o equal to or less than zero is always greater than unity and approaches unity asymptotically as the altitude is increased indefinitely. Reference 1 shows, however, that the real annular nozzle with θ_o equal to zero follows this theoretical trend only up to a certain altitude (of the order of one or two diameters in the experiments of Reference 1), called the critical altitude, at which point the augmentation factor suddenly drops to a value appreciably less than unity and remains less than unity (and essentially constant) at all higher altitudes. The explanation of this behavior lies in the turbulent mixing of the jet.

It was mentioned previously that the real jet tends to entrain air from the base cavity. At low altitudes, this tendency is satisfied by a shallow impingement of the jet on the ground, which feeds part of the jet air back into the cavity to replace the air entrained into the jet. At high altitudes, on the other hand, even if atmospheric pressure were somehow maintained within the base cavity, the turbulent expansion of the jet would seal off the base cavity long before the jet reached the ground.
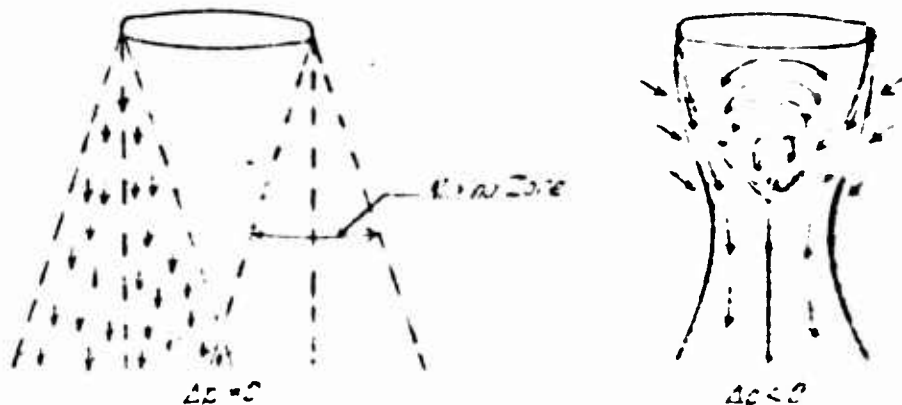
Annular Nozzle at High Altitude

Once this happens, the entrainment of air from the cavity will tend to create a vacuum, and, under the influence of a negative Δp, the jet will curve inward and converge, at some angle θ_c, to a stagnation point at the axis of the nozzle. From this point, part of the jet air will be deflected upward into the cavity, thus replacing the air removed by entrainment, and the rest will be deflected downward along the axis of the nozzle in the form of a "solid" (nonannular) round jet. An equilibrium is reached when the Δp has fallen sufficiently low to curve the jet sharply enough so that the impingement angle, θ_c, is sufficiently steep to deflect as much air back up into the cavity as is being entrained from it.

The equilibrium value of Δp depends upon the mixing properties of the jet flow and is thus not readily susceptible to calculation. If Δp (or A) is given, the jet path can be calculated

as before, for a pseudo-ideal jet, by equating the static
pressure difference to the centrifugal pressure and ignoring
the inconsistency involved in having fluid deflected back into
the base cavity (or making some suitable assumption to account
for it). The equations and the form of the solution are the
same as before. The quantity of interest is x_c, the distance
of the stagnation point below the nozzle.
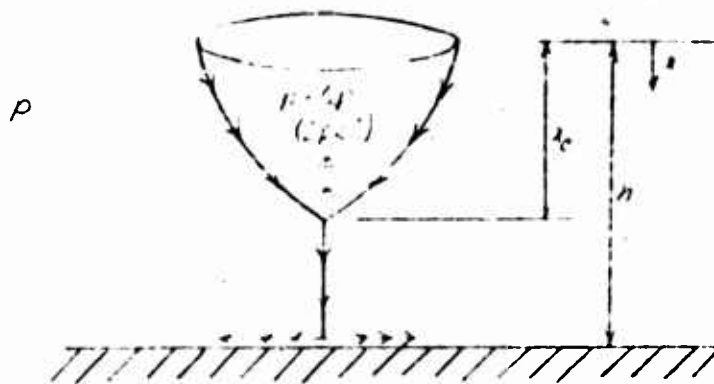
It seems reasonable to suspect that x_c is of the same order of
magnitude as the critical altitude, h_c, since the type of flow
under consideration can exist only if h is greater than x_c.
This quantity is given by

$$x_c/r_0 = \frac{1}{2\sqrt{\cos\theta_0 - A}} \int_{\theta_0}^{\cos^{-1}A} \frac{\cos\theta\, d\theta}{\sqrt{\cos\theta - A}}$$

Some results of numerical integration of this formula, for θ_0
equal to zero, are presented in Figure 6. Also indicated in
Figure 6 are the approximate ranges of critical altitude,

h_c/x_o, and of augmentation factor, for $h > h_c$, encountered in
the tests of Reference 1. It is seen that the experimentally
determined ranges are quite consistent with the notion that

$$x_c \approx h_c$$

This is of little more than academic interest, of course, since,
even if this relationship were rigorously substantiated, experi-
ment would still have to be relied upon for values of A for
$h > h_c$, and so one had just as well rely upon experiment for
values of h_c directly.

An interesting feature of the above concept of the jet
behavior near the critical altitude is the apparent likelihood
of a "hysteresis" effect in the curve of A versus h. It would
seem likely that the discontinuity in this curve would occur at
a higher altitude when the nozzle is raised slowly from near the
ground than when it is lowered slowly from a considerable height
(as considered here and in Reference 1).
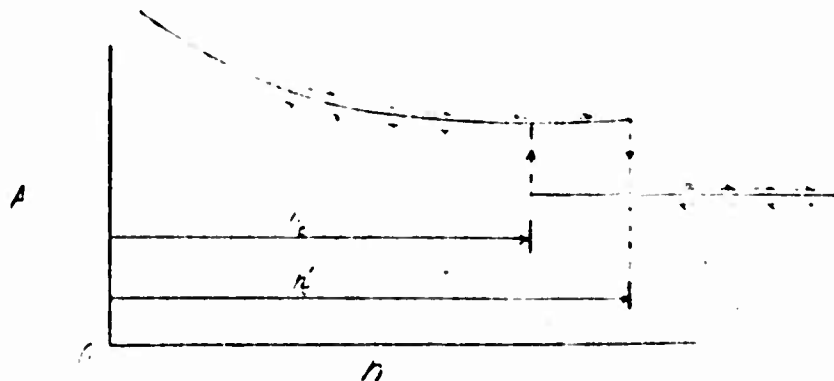
Reference 1 neither confirms nor denies the existence of this effect. It will be interesting to observe whether such an effect is revealed by future experiments.

CONCLUSIONS

Calculations based on the assumption of a thin, nonmixing jet are found to give results in reasonable agreement with available experimental results and will probably give reasonable results over most of the range of practical interest. The errors incurred in neglecting the jet mixing become important when the nozzle is at high angles of incidence to the ground and when the nozzle is at altitudes near the critical altitude. At altitudes above the critical altitude, the mixing plays a primary role in determining the flow pattern, but certain features of the flow can still be correctly represented by ideal-jet calculations.

Aerodynamics Laboratory
David Taylor Model Basin
Washington, D. C.
July 1957

APPENDIX

APPROXIMATE ANALYSIS OF EXPERIMENTAL DATA

FROM REFERENCE 1

The results presented in Reference 1 are presented largely in terms of empirical parameters. In order to convert these results into a suitable form for comparison with the theory (i.e. into augmentation factor versus h/r_o) with a minimum of labor, several simplifying approximations were employed:

1. Figure 10e of Reference 1, presenting (in the nomenclature of Reference 1)

$$\frac{F_t}{A_j P_j} \left(\frac{A_j}{A_t}\right)^{0.15} \left(\frac{P_n}{P_o}\right)^{0.20} \quad \text{versus} \quad \frac{h}{D_b}$$

was assumed to represent adequately all of the experimental data on which it was based.

2. The jet momentum was assumed to be adequately approximated, over the range covered, by

$$J = 1.67 \, A_j \, P_{j_{av}} \left(\frac{P_n}{P_o}\right)^{-0.20}$$

where $P_{j_{av}}$ is the difference between the jet total pressure, P_n, and the average static pressure within the free jet. This expression, if applied to a jet with uniform static pressure, implies a nozzle loss (compared to the theoretical momentum) ranging from 13 percent at a pressure ratio of 1.2 to 4 percent at a pressure ratio of 2.6.

3. The average static pressure within the free jet was assumed to be the arithmetic mean of the pressure within the cavity and the atmospheric pressure outside the jet. Thus

$$P_{j_{av}} = P_j \left(1 - \frac{F_b}{2A_b P_j} \right)$$

where F_b/A_b is equivalent to the "Δp" employed in the present report. The required values of $F_b/A_b P_j$ were taken from Figure 8 of Reference 1.

With these approximations, the augmentation factor, A, was calculated from

$$A = \frac{F_t}{J} = \frac{\left[\left(\frac{F_t}{A_j P_j} \right) \left(\frac{A_j}{A_t} \right)^{0.15} \left(\frac{P_n}{P_o} \right)^{0.20} \right]}{1.67 \left(\frac{A_j}{A_t} \right)^{0.15} \left(1 - \frac{F_b}{2A_b P_j} \right)}$$

This result implies that, within the range covered by Figure 10e of Reference 1, the augmentation is, for practical purposes, independent of the nozzle pressure ratio.

Reference 1 presents sufficient information to permit the results of the tests of three nozzles (nozzles A, B, and C having nozzle widths of 0.101 D_b, 0.070 D_b and 0.053 D_b, respectively) to be reduced by the above formula. Upon performing the calculations, it was found that the results for the three nozzles could be represented (within about ±2 percent) by a single curve of augmentation factor versus h/D_b.

The base area, $\pi D_0^2/4$, of Reference 1 corresponds to the disc area πr_c^2 of the present report. The correspondence in altitude parameter is thus

$$h/r_0 = 2\ h/D_0$$

REFERENCE

1. Von Glahn, Uwe H. Exploratory Study of Ground Proximity Effects on Thrust of Annular and Circular Nozzles. Wash., Apr 1957. 48p. incl. illus. (National Advisory Committee for Aeronautics. TN 3982)
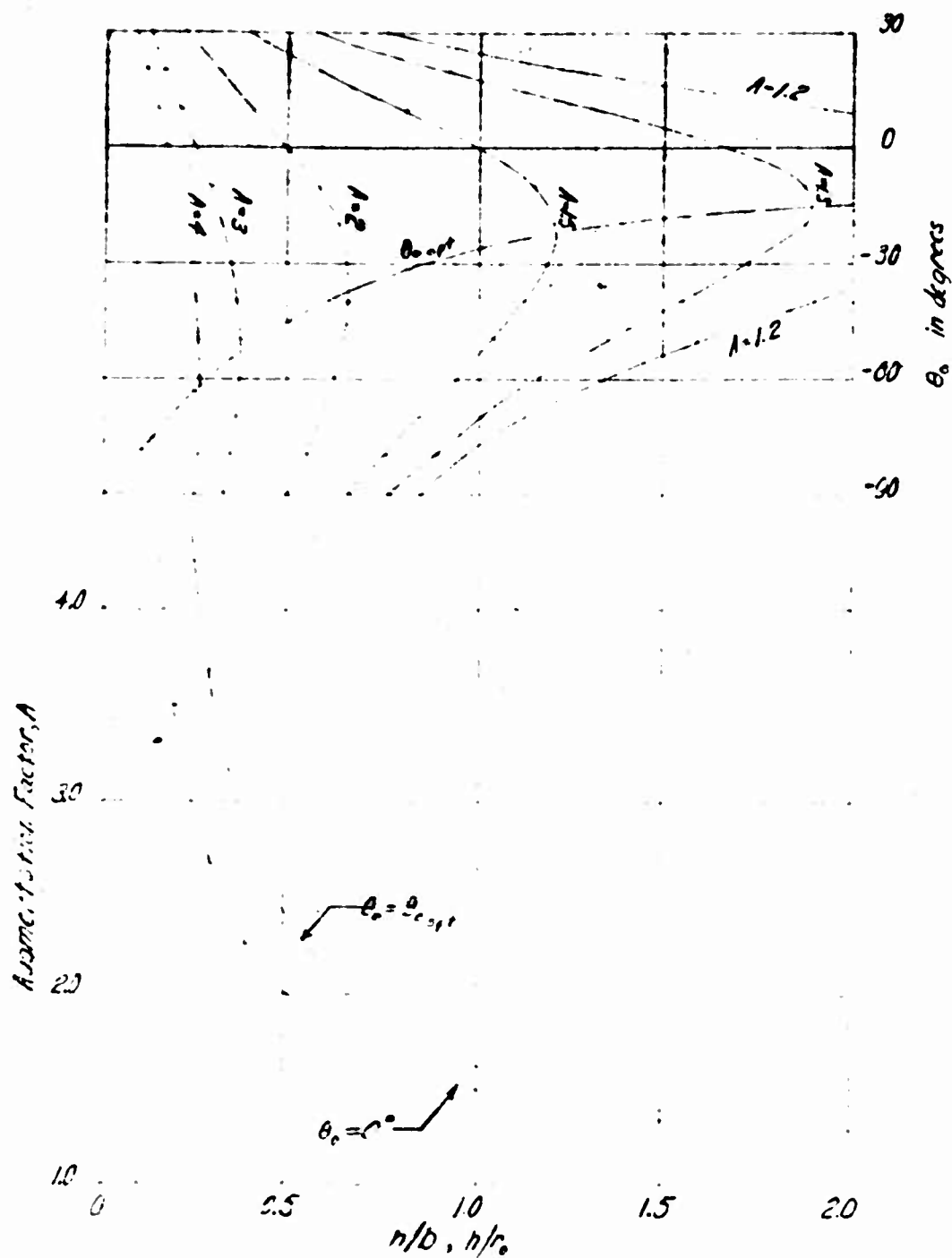
Figure 1 - Augmentation Factor According to a First Two-Dimensional Solution and an Approximate Analytic Approximation

$$\text{Augmentation Factor} = 2.414$$

$$\theta_0 = -\frac{\pi}{4}$$

$$\beta = \frac{1}{2(2.414-0.707)} = 0.293$$
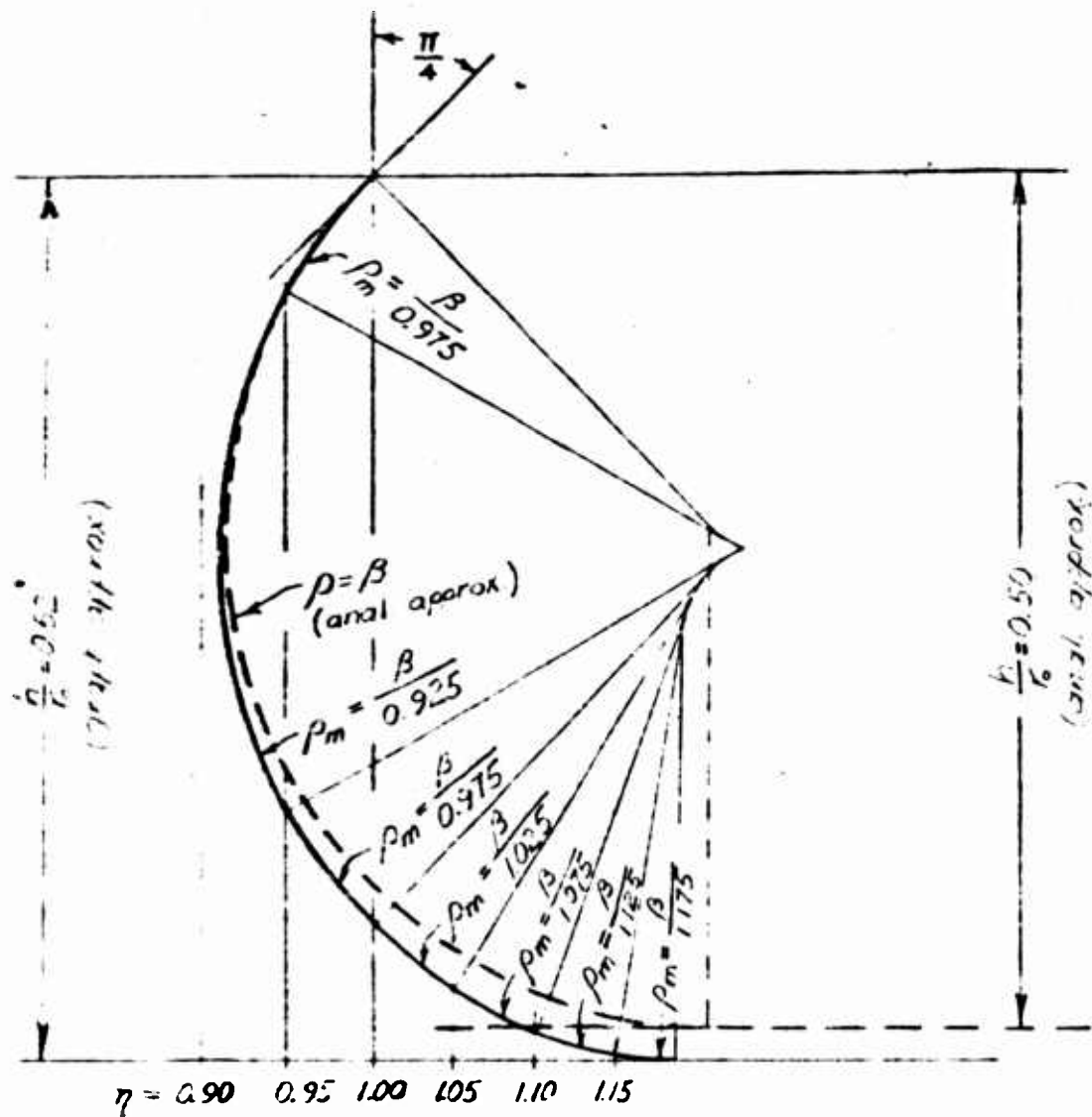
Figure 2— Illustrative Example of the Graphical Approximation of the Jet Path for an Annular Nozzle With Ground-Effect Augmentation

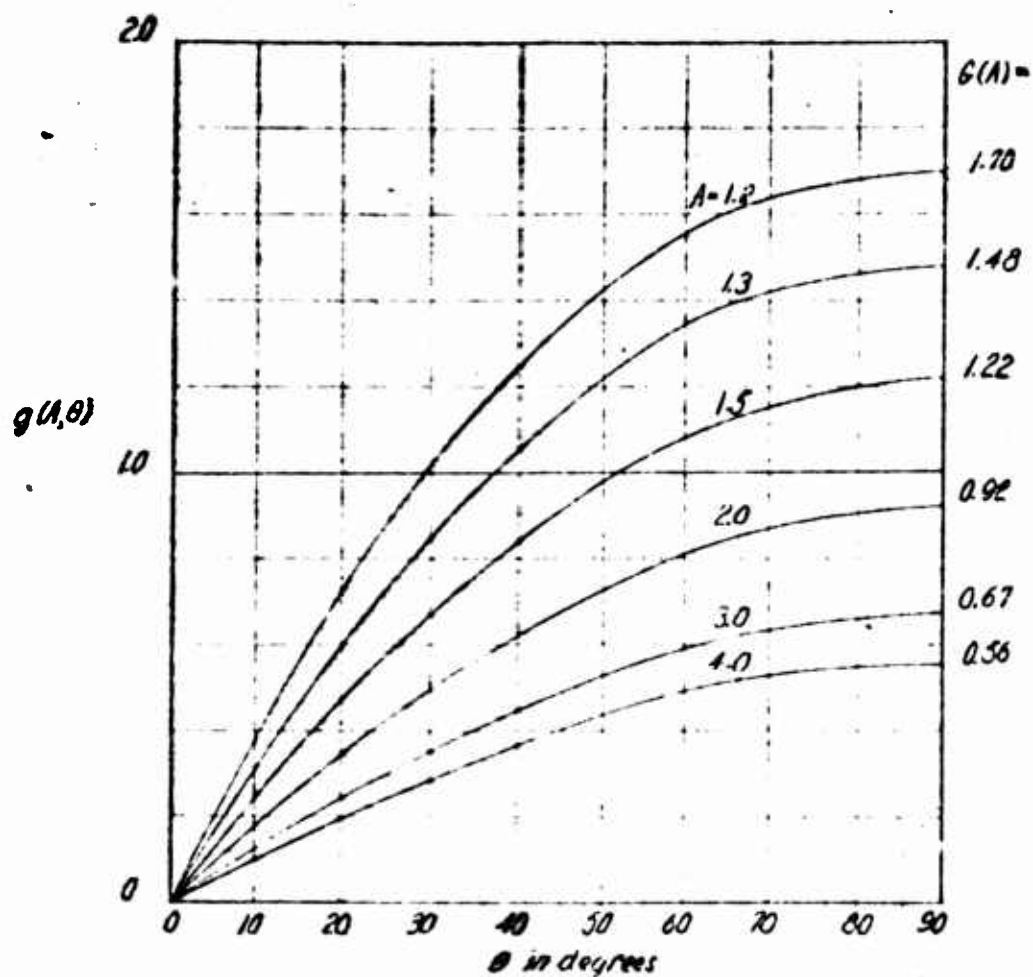Figure 3 - Some Values of the Function

$$g(\lambda,\theta) = \int \frac{\cos\theta\, d\theta}{\sqrt{\lambda - \cos\theta}}$$

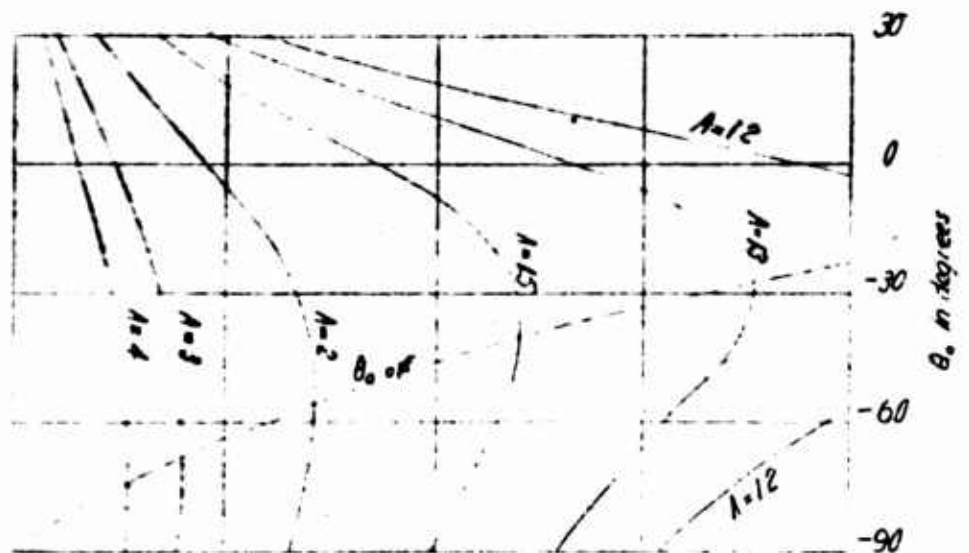$$G(\lambda) = \int_{0}^{\frac{\pi}{2}} \frac{\cos\theta\, d\theta}{\sqrt{\lambda - \cos\theta}}$$
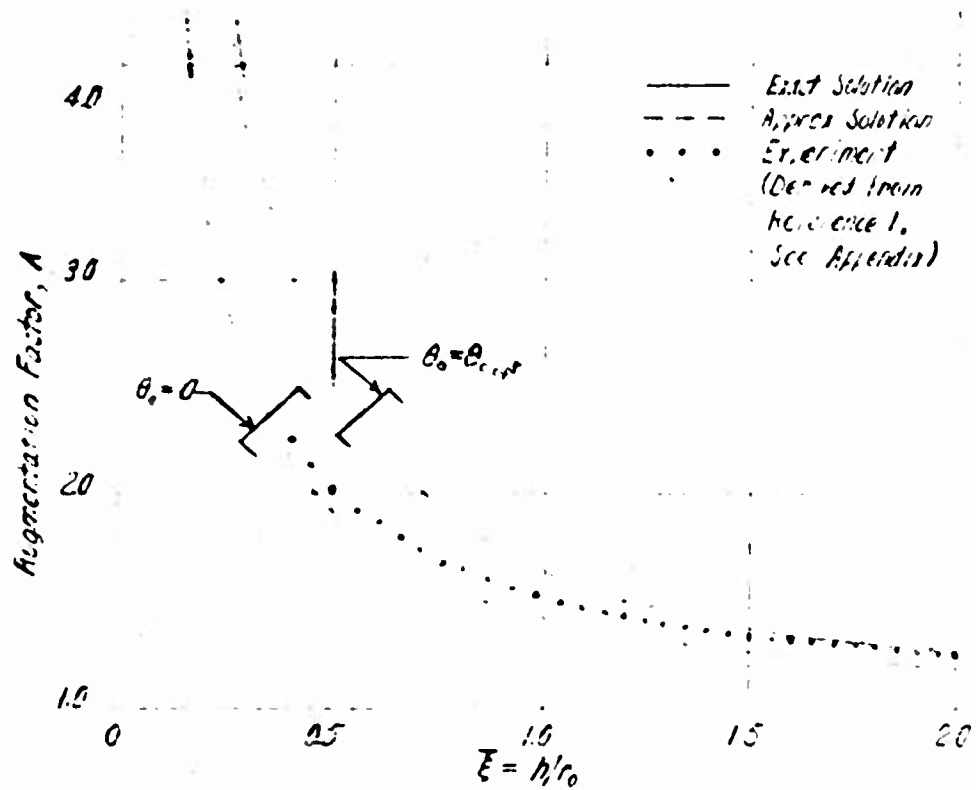
Figure 4 – Augmentation Factor for an Axially-Symmetric Annular Nozzle
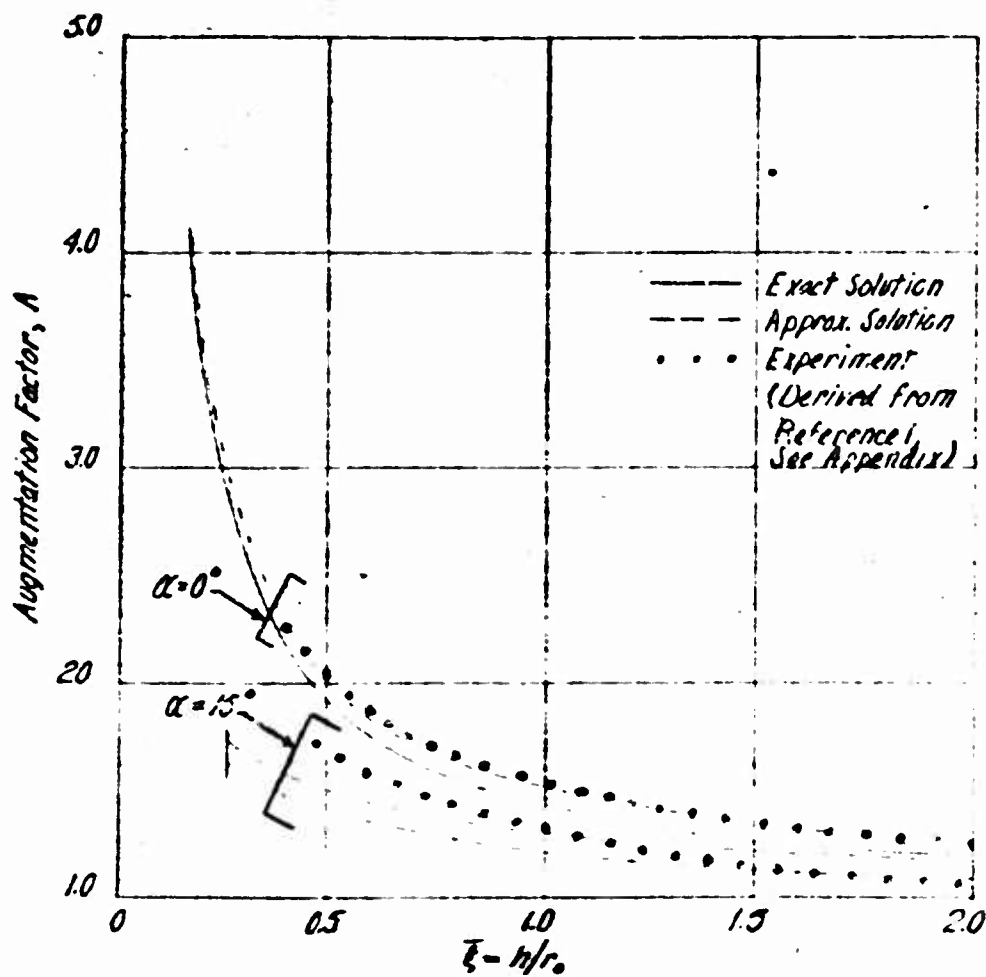
Figure 5 - Effect on A of 15° Incidence to the Ground

$\theta_o = 0°$

①, ② - Approximate ranges of r_t/r_c and A $(h > h_c)$ observed in Reference 1

②
①
4
3
2
1
0
$\frac{r_t}{r_c}$
$\frac{r_t}{r_c}$
0
0.5
1.0
Augmentation Factor, A

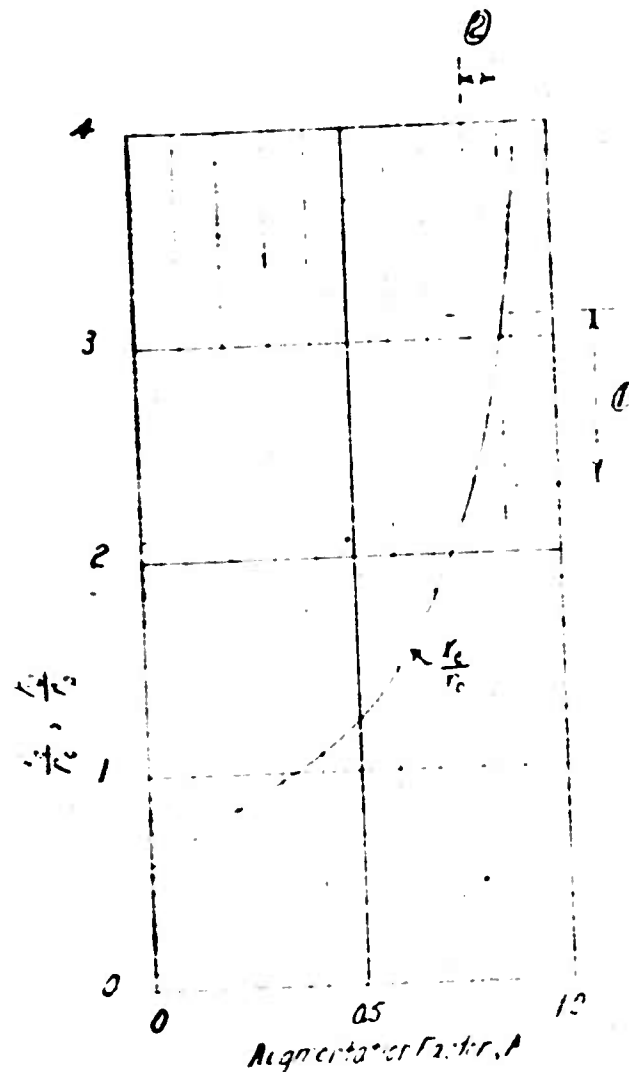

Figure 6 – Theoretical Variation of r_t/r_c With Augmentation Factor for an Ideal Axi-Symmetric Nozzle at Altitudes Above the Critical Altitude

$\varepsilon = 0°$

FIGURE 6

[illegible]

Copies

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WASHINGTON 7, D.C.

Errata

to

Aero Report 923

by

Harvey R. Chaplin

July 1957

Pen and ink changes:

Page 28: Change "A" to "2A - 1" throughout the equation.

Page 39, Fig. 6: Change abscissa label "Augmentation Factor A" to "2A - 1".

Change range "② $0.8 < A < 0.88$" to range "② $0.60 < (2A - 1) < 0.76$".